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A mechanism of crack branching in polymethyl methacrylate and the origin of the bands on the surfaces of fracture

MICHAEL J. DOYLE*
Department of Mechanical Engineering, University of Pittsburgh, Pittsburgh, PA 15261, USA

At low crack velocities the fracture of high molecular weight polymethyl methacrylate occurs by the separation of a thin craze layer ahead of, and coplanar with, the propagating crack tip. Above some critical velocity, about 400 m $\sec^{-1}$ at room temperature, craze branching or bifurcation is initiated. The craze branching does not cause any detectable surface roughening of the fracture surface until the crack tip stress is sufficient to initiate cracks in the craze branches. At this stage the formation of the branching craze-cracks causes surface roughening (bands or striations), a deceleration of the main fracture and a drop in the stress amplitude around the crack tip which is below that necessary to initiate branching crazes. The fracture then reverts back to the simpler mechanism, with no surface roughening. The repetition of this process gives rise to the banded appearance of the fracture surface.

1. Introduction

Increasing roughness of the surface of fracture and crack branching at high crack velocity are common features in the fracture of most brittle materials. In glassy thermoplastics, however, the beginning of the roughening usually occurs in the form of alternating rough and smooth bands which lie perpendicular to the direction of crack propagation. Prosvirin [1] has ascribed these bands to the same cause as that of the periodic markings on the cleavage fracture surfaces of some inorganic crystals such as lithium fluoride (LiF). A cleavage wedge moving with constant velocity may cause oscillations in the velocity of the crack which precedes it if the fracture surface energy of the material decreases with crack velocity [2–4], giving rise to "stick-slip" behaviour and periodic markings or hesitation lines on the surface of fracture. But in general the fracture surface energy of polymethyl methacrylate (PMMA) and polystyrene increases with the crack velocity and, unlike the above mentioned effects in LiF, the bands are formed over a wide range of temperature, specimen geometry and loading conditions. The only necessary condition, it seems, is that the crack velocity must be above some critical value.

An explanation for two related forms of bands observed on the surfaces of fracture of polystyrene (M_v about 2×10^5) was described previously [5, 6]; the same explanation also applied to the fracture bands in styrene–acrylonitrile copolymer and polymethyl methacrylate of low molecular weight (M_v about 10^5), but not for cast polymethyl methacrylate of high molecular weight (HMW–PMMA, M_v about 10^6). The mechanism of formation of the bands (ribs or striations) in HMW–PMMA has been the subject of several recent papers but the usual explanations, the so called Yoffe crack branching effect, Wallner lines, or secondary microcrack nucleation, are not the primary cause of the bands in this case, just as they are not for the bands in polystyrene either. Since HMW–PMMA is frequently used as the exemplar of a brittle material, especially with respect to dynamic crack propagation studies, it is of interest to understand the actual mechanisms of fracture.

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*Present address: Exxon Chemical Company, Elastomers Technology Division, P.O. Box 45, Linden, NJ 07036, USA.

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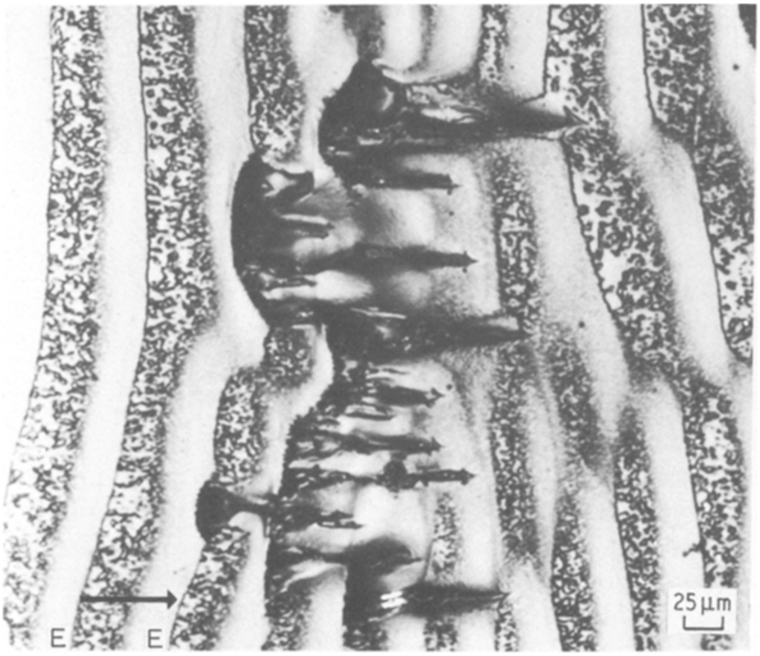

Figure 1 Surface of fracture of polystyrene cleaved at 80° C showing Type I bands and multiple layered Type II bands at the centre of the micrograph using reflected light.

Observations of the fracture surfaces of HMW–PMMA show that the mechanism of fracture changes at some critical velocity (about 400 m sec^{-1} at room temperature and corresponding to a critical stress level at the crack tip) from propagation of a single craze wedge to the nucleation of branching crazes which is effectively a trifurcation of the craze. The resulting increase in the rate of energy absorption (with respect to the increasing crack length) decelerates the crack; the branch craze cannot be extended, and the velocity soon drops below that critical value necessary for the immediate initiation of new branching crazes. At the lower velocity the fracture mechanism reverts to its previous, simple mode, namely, the propagation of a single craze wedge. The bands arise from oscillations between the two different mechanisms of fracture. In the latter sense, the effect is similar to that in the other glassy polymers previously mentioned but the mechanism of nucleation of the band of multiple crazing is quite different, as is evident from the different morphologies of the fracture surface. For the sake of clarity, the mechanism of formation of the bands in polystyrene will be reviewed before presenting the new observations which reveal the mechanism of nucleation of the bands in HMW–PMMA as well as the mechanism of crack branching in this material.

2. Formation of striations by craze wedge detachment

This mechanism is demonstrated in its simplest form by the fracture surface of polystyrene at 80° C as shown in Fig. 1 and illustrated schematically in Fig. 2. A crack in polystyrene is preceded by a layer of craze, an expanded form of the polymer of density less than half that of the bulk polymer. At low speeds the crack propagates by viscoplastic separation of the craze layer along its median plane while the craze tip propagates into the bulk at the same speed (Fig. 2a). As the crack accelerates the craze stress increases until, at some critical value, the craze abruptly fractures at its boundaries with the adjacent bulk in a low energy, quasi-brittle fracture (Fig. 2b). The fracture criterion here is probably a critical stress: the strength of the craze–bulk boundary. The brittle fracture does not occur in the bulk proper, for if it did there seems to be no reason why, once started, it would stop. But the fracture does not extend beyond the edge of the craze at its tip, E. The boundary fracture must occur at some structural weakness near the craze boundary layers. As the craze wedge peels off rapidly from the bulk, the stress ahead of the new crack tip increases (Fig. 2b) and a new craze (plastic zone) is formed ahead of it at a high strain rate (Fig. 2c). If the boundary conditions remain the same

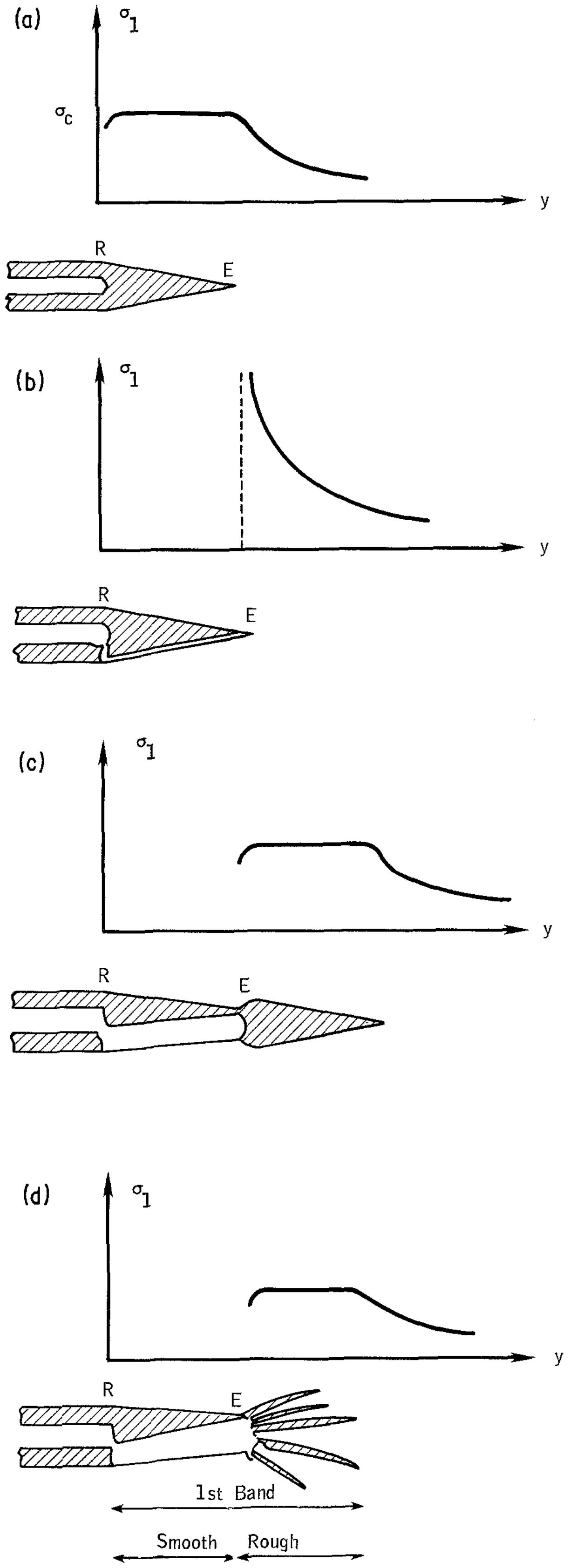

Figure 2 Schematic diagrams of formation of fracture bands or striations by craze wedge detachment: (a) craze propagation along the median plane of the craze, (b) craze detachment at one craze-bulk boundary, (c) formation of a single, new craze wedge directly ahead of and coplanar with the previously detached craze, (d) formation of multiple crazes and cracks radiating from the tip of the detached craze wedge. Also shown is a plot in each case of the principal stress acting in the direction perpendicular to the plane of the craze along the craze and in the bulk directly in front of it.

during this crack "jump" then the new craze should grow to about the same length as the old one if the craze length is much less than the length of the crack. But since the craze stress is dependent on the strain rate, the new craze, being formed at a higher strain rate, reaches the critical stress when it is shorter and thinner than the quasi-static craze which preceded it when the first craze instability occurred.

Subsequent craze instabilities all occur under similar conditions at high local strain rates (nominally about 10^6 sec^{-1}), in what is macroscopically a fast fracture ($\dot{c}$ about 100 m sec^{-1}). The bands are then more or less of uniform width. Each band is a regenerated craze wedge which fractures at a structurally weak plane at one of its boundaries with the adjacent bulk polymer. Sometimes the detached craze wedge lies completely on one surface of fracture or the other, as is often the case with polycarbonate (Fig. 3) for example. In polystyrene the wedge is usually broken up into fragments because the craze boundary fracture is initiated at many different sites on both craze–bulk boundaries [1].

This mechanism of the formation of bands, which will be called Type I, is not dependent on stress waves nor any mechanism of crack branching. The several difficulties which arise in an explanation based on the interaction of stress waves with the propapating crack, as in the formation of the familiar "Wallner lines", have been discussed by several authors [5, 7, 8] and will be mentioned again in Section 4.

The bands observed on the fracture surfaces of polystyrene broken at room temperature, shown in Fig. 4, are usually much rougher than the Type I bands but they are clearly related, since the finer bands in between are Type I. The difference is that instead of a single craze wedge developing ahead of and coplanar with the preceding detached craze wedge, a bundle of crazes and cracks radiate from the line EE, which marks the tip of the detached wedge [5, 6]. As before, the brittle craze boundary fracture cannot propagate beyond the craze edge, but instead of a single new craze layer, a bundle of crazes is formed (Fig. 2d). There are two reasons for this. Firstly, if the craze detachment occurs more rapidly than craze growth, the stress in a region ahead of and around E (Fig. 2b) may be high enough to nucleate several crazes before any one craze can propagate far enough to restore the crack tip region into

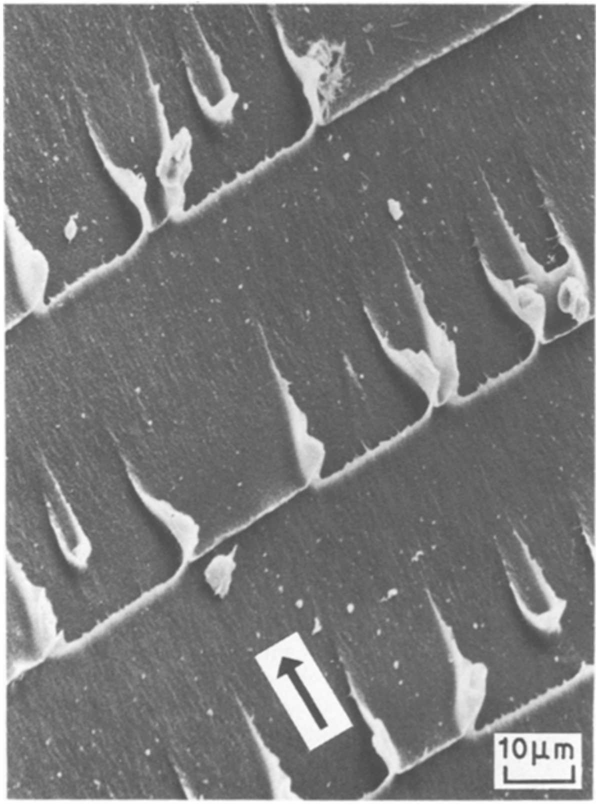

Figure 3 Scanning electron micrograph (SEM) of the surface of fracture of polycarbonate showing successive bands of single craze wedge detachment.

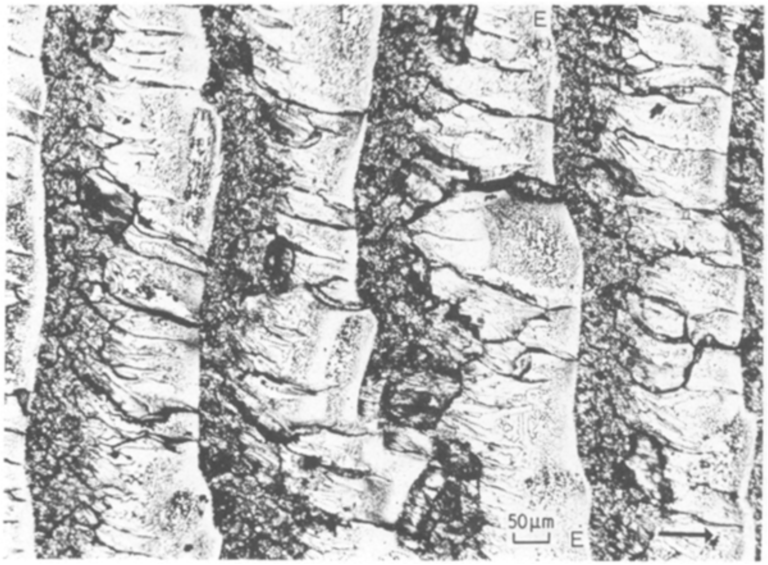

Figure 4 Fracture surface of polystyrene formed at room temperature and showing Type II bands of multiple crazing using reflected light.

(dynamic) equilibrium with the applied loading. Secondly, the sudden detachment of the craze causes the newly formed and traction free walls of the crack and the region immediately surrounding its tip, E, to accelerate. Multiple crazes and cracks may result from the inertial forces in the same way that the case of an exploding shell bursts into many fragments [9]. The energy absorbed by the band of multiple crazing is released by the change in fracture mechanism immediately preceding the initiation of the band. For energetic reasons, which will be discussed in more detail below, only one of the crazes emerges from the bundle. It accelerates, fractures and the process is repeated. The fundamental cause of the bands is the same as before: glassy thermoplastics cannot fracture without first forming a craze, but the latter can fail by any one of several different mechanisms, each of which can have quite different energy absorption characteristics. If, at increasing crack speed, the fracture mechanism suddenly changes to another that has a larger energy absorption, then the crack is decelerated and the fracture mechanism reverts to its previous form, thus setting up oscillations between the two mechanisms. Bands formed by multiple cracks and crazes will be called Type II bands.

The size of the shatter band should be related to the amount of elastic strain energy released by the sudden detachment of the preceding craze wedge. From the principle of virtual work that is given by

$$W = \int_0^R \sigma_c U_c \, dr \qquad (1)$$

where σ_c and U_c are the stress and elastic displacement across the craze and R is the critical craze length. Each of these quantities is dependent on molecular weight so that the sensitivity of the band spacing to polymer molecular weight (8) is to be expected.

As was mentioned in Section 1, it has been proposed that Types I and II bands arise from dynamic effects. Since these same theories are suggested also for the mechanism of banding in HMW–PMMA, their discussion will be delayed until the morphology of the bands in this material is presented. However, it is appropriate to remark here than in polystyrene the first band can be initiated at very low crack velocities [6], under quasi-static conditions, where dynamic effects are not of any significance, whereas in HMW–PMMA fractured at room temperature the bands only appear at crack speeds of about 400 m sec^{-1} so that dynamic effects cannot be ignored.

3. Formation of fracture bands by craze branching

Figs. 5 and 6 show the fracture surface of a tensile specimen of HMW–PMMA ($M_v = 10^6$) in the region of fast crack propagation (about 400 m sec^{-1}). Although there is the obvious similarity of rough and smooth bands as in polystryene, there are important differences in the detailed

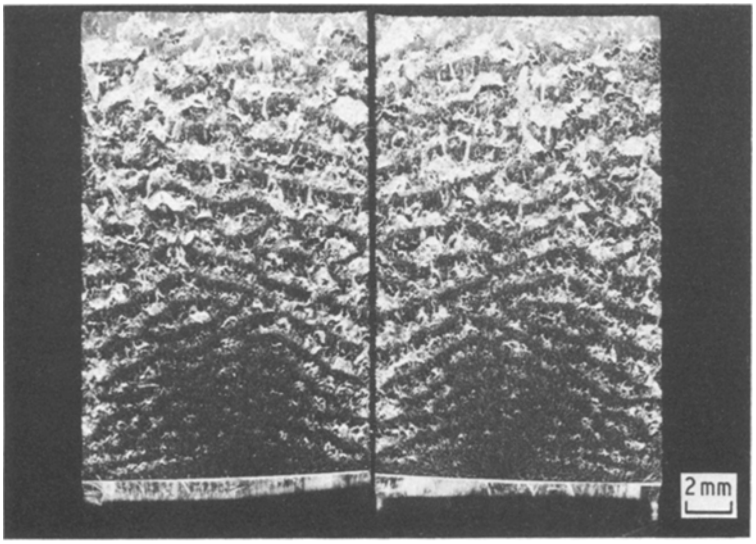

Figure 5 Surface of fracture of cast polymethyl methacrylate (PMMA), notched bars (2.5 cm × 1.27 cm, 0.4 cm notch depths) fractured in tension, showing "fracture bands/striations" and intersecting bands with a cross hatch pattern at the middle of the fracture surface using reflected light.

morphology of the smooth band. In polystyrene the smooth band corresponds to the fracture of the craze along its boundaries with the bulk. But in HMW–PMMA the similarity of the optical interference patterns on matching surfaces of fracture in the smooth bands shows that the craze has separated more or less along its median plane. This mechanism of craze fracture is qualitatively not different from that which occurs at high speed but before the banding begins. Another difference is that there is no characteristic sharp boundary line which is polystyrene clearly separates the smooth band from the beginning of the following rough band (compare, for example, Figs. 4 and 6).

Given that the rough band of multiple cracking and crazing can be formed without leading immediately to macroscopic branching, it must decelerate the crack, just as is the case in polystyrene. At the lower velocity the fracture reverts to the simpler single craze-crack configuration, followed by acceleration of the crack and formation of the next rough band. What has to be explained is the mechanism of the initiation of the roughening and why macroscopic branching in PMMA is preceded by an extensive region of banding in which the rough bands are themselves of increasing roughness. In glass the surface roughening increases continuously up to the point where macroscopic branching occurs; there is no banding.

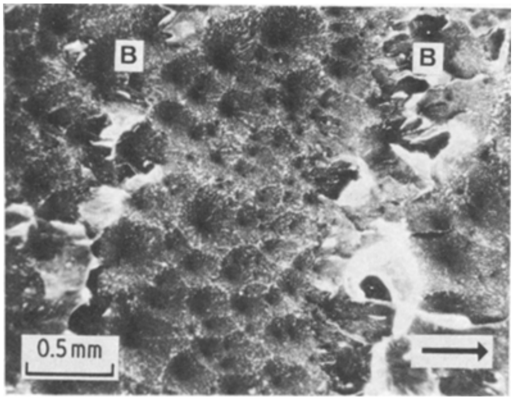

Figure 6 Bands in PMMA, B; the dark spots in the area between the bands are the foci of the fracture parabolas using dark field illumination.

Fig. 7 shows the surface of fracture cut from a HMW–PMMA plate which was originally 200 cm x 63 cm x 0.64 cm and containing a sharp edge crack (arrow) of 0.4 cm length halfway along a longer side. The plate was fractured in tension. These specimens, which were tested more than twenty five years ago by Busse *et al.* [10], still show the characteristic interference colours (now somewhat faded) which arise from the craze layer on the fracture surface. On these fracture surfaces the characteristic changes in the morphology with increasing crack speed occur gradually over a length of more than 40 cm (before the crack branches) and the details can be easily resolved. The crack velocities referred to in the following discussion and shown in Fig. 8 are taken from the paper by Bulaney and Brace [11] who analysed the original velocity measurements made by Busse *et al.* (using the ladder resistance grid method). A correlation of fracture surface morphology and crack speed has also been given by Kobayashi *et al.* [12]. Fig. 6 shows the region

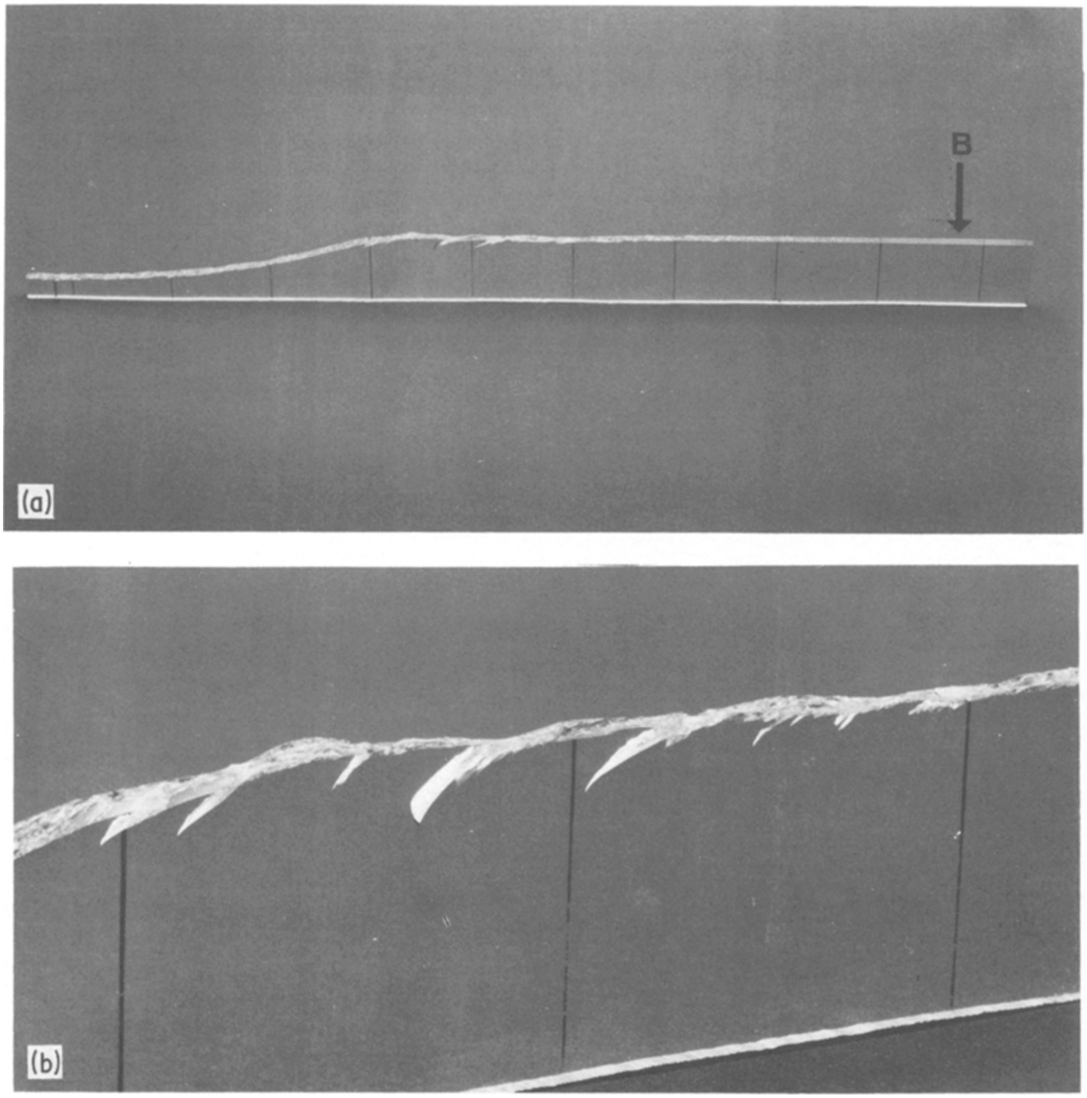

Figure 7 Macrophotographs of the surfaces of fracture cut from a PMMA plate (200 cm × 63 cm × 0.64 cm) which was edge notched and fractured in tension.

between B and D of Fig. 8 (about 400 to 500 m sec^{-1}) in which there is a transition from a smooth surface, in which fracture occurs continuously in a single plane, more or less in the centre of the craze layer, to the onset of the fracture bands within which multiple crazes and cracks occur above and below the main surface (that level defined by the smooth bands in between the rough bands). The roughening of the surface profile occurs quite suddenly which is in contrast to the gradually increasing roughness in the mist region on glass fracture surfaces. Blocks of polymer protrude from the surface of fracture with corresponding holes about 50 μm in depth on the opposite, mating surface of fracture. Fig. 9 shows the region B from Fig. 7 at high magnification and viewed obliquely, in indirect illumination, from the underside of the fracture surface, i.e. through the specimen. The dark patches are crazes branching down from the surface of fracture (bright) and forwards in the direction of crack propagation (arrow). Fig. 10 shows the same surface of fracture viewed normally in dark field illumination (at grazing incidence from the underside of the fracture surface and directed towards the origin of fracture). The bright lines

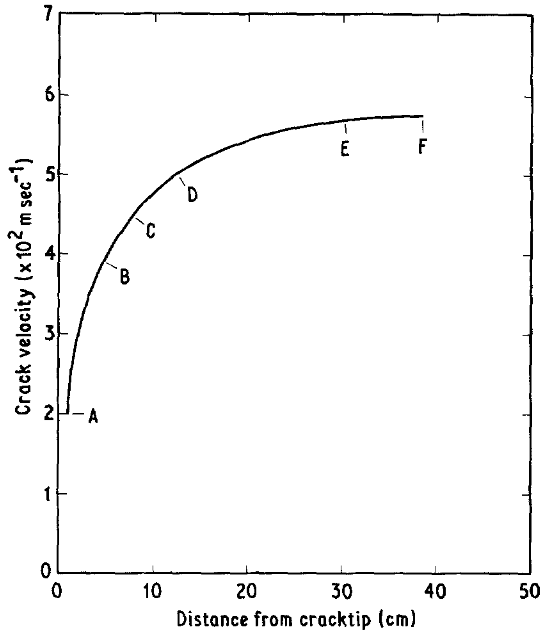

Figure 8 Crack velocity versus crack length for an edge notched plate fractured in tension (data from [11]). Branching crazes begin at conditions corresponding to B, fracture surface roughening at C, and macroscopic branching at F.

lying perpendicular to the direction of crack propagation mark the locus of intersection of the branching craze with the surface of fracture. The branching crazes cannot be seen when viewed in normally incident bright field illumination. Fig. 11 shows a cross section of the fracture surface shown in Fig. 10.

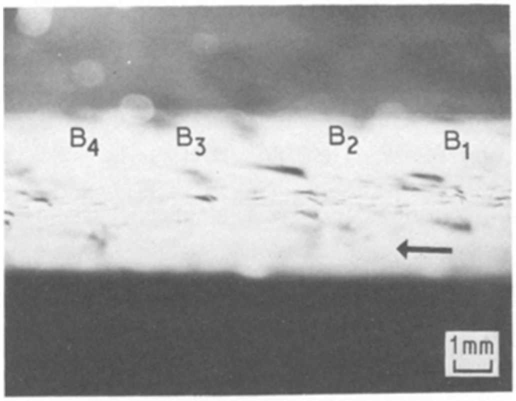

Figure 9 The surface of fracture of the specimen shown in Fig. 7 viewed through bulk beneath it. The branching crazes appear as dark streaks (arrows). Oblique illumination was used.

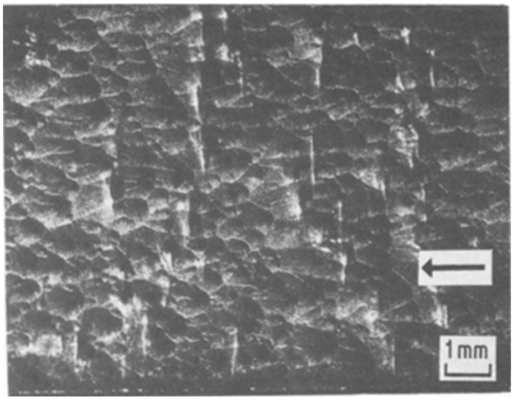

Figure 10 Surface of fracture from Fig. 9 using dark field illumination. Bright lines transverse to the direction of crack propagation mark the intersection of the branch craze with the surface of fracture.

This is a characteristic and reproducible feature of crack propagation above a certain crack velocity in HMW–PMMA (about 400 m sec^{-1} at room temperature) which precedes the region containing the more obvious bands of striations. The same phenomenon can be observed occasionally, however, in low velocity crack propagation in HMW–PMMA (about 10^{-3} m sec^{-1}) and also, although quite rarely, in polystyrene at similarly low crack speeds. Then the phenomenon can be easily observed and followed in the microscope during slow cleavage crack propagation. Fig. 12a shows such a branching craze in polystyrene as it was observed where the plane of fracture (crazing) intersected the side surface of the specimen. Fig. 12b shows the same phenomenon in HMW–

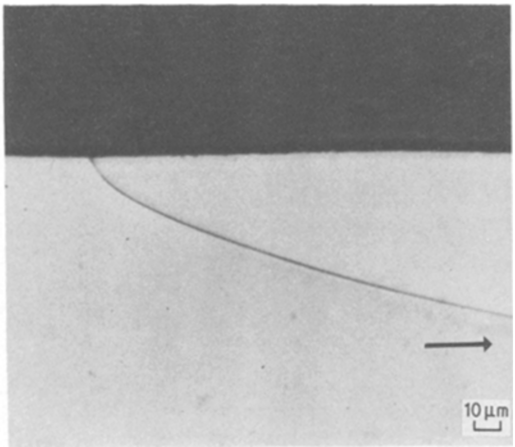

Figure 11 Cross section of a PMMA fracture surface from the region B of branching crazes using normal incident light.

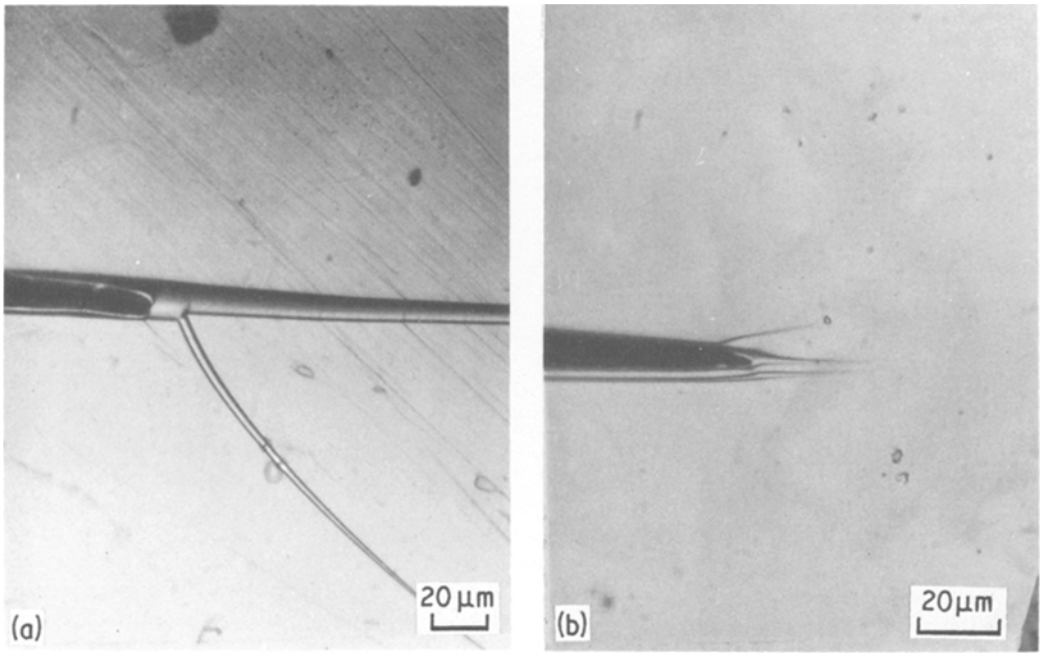

Figure 12 Branching crazes observed in slowly propagating cleavage cracks in side view at the intersection of the plane of the crack with the side surface of the plate specimen; (a) polystyrene, (b) cast PMMA. Reflected light was used.

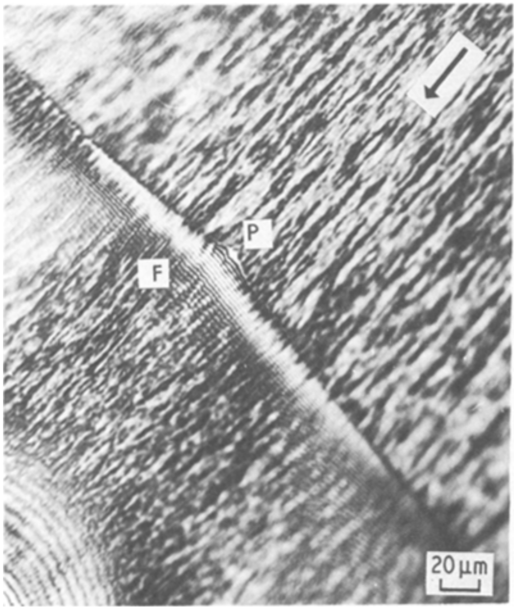

Figure 13 Branching craze in the surface of fracture of PMMA from the region of slow (less than 1 cm $\sec^{-1}$) crack propagation. The branch craze originates from a small advance fracture P (just visible on the original print). Reflected monochromatic light, $\lambda = 546$ nm, was used.

PMMA in the slow crack velocity region. The branching crazes initiate at a point on one boundary of the main craze, usually away from the edge of the specimen, where an advance fracture (fracture parabola) has nucleated at the craze–bulk boundary [5]. This is evident from the final fracture surface of PMMA shown in Fig. 13; the nucleating defect is P and the closely spaced interference fringes, F, arise from the wedge of bulk polymer lying between the surface of fracture and the branching craze below it. The distribution of stress in the bulk polymer adjacent to the craze is such that the minor principal stress (in the plane of the specimen), which is in a direction approximately parallel to the craze–bulk interface, is not much less than the larger principal stress which acts across the craze. The presence of a hole in the craze is sufficient to rotate the direction of the maximum principal stress in the adjacent bulk through an angle $\pi/2$ and that stress then exceeds the craze initiation stress at the craze boundary. The branching craze initiates and propagates in a direction almost perpendicular to the boundary of the main craze but curves forwards in the direction of crack propagation as the stress

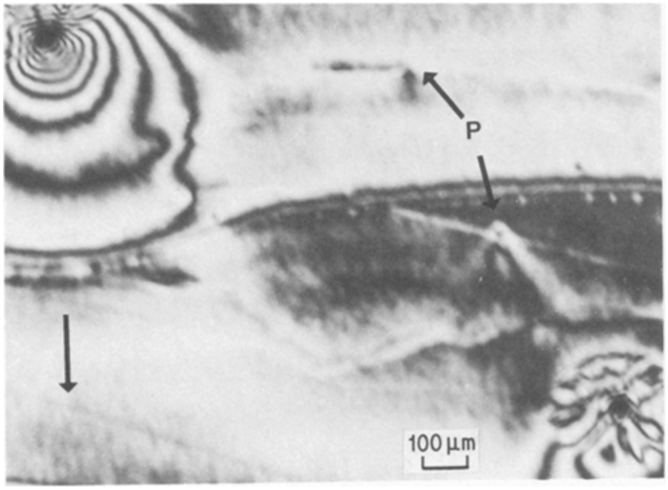

Figure 14 Branch crazes on polystyrene fracture surfaces formed by slow cleavage crack propagation. Branch crazes originate from small advance fractures, P, which are very much smaller than the fully developed advance fractures not associated with branching crazes. Reflected monochromatic light, λ = 546 nm was used.

field of the main craze-crack moves forwards with the propagation of the crack.

The branching crazes in the high velocity region shown in Fig. 10, however, are not so obviously associated with the advance fractures which are clearly seen in the special cases shown in Fig. 13. This is probably because the initiation of the branching craze inhibits the subsequent growth of the advance fracture itself. Some evidence for this hypothesis was observed on the fracture surface of polystyrene at low crack speeds, see Fig. 14, where two small branching crazes have nucleated at advance fractures, P, which are very much smaller than those adjacent to it.

The branching craze, once initiated, rapidly grows outwards to a distance roughly equal to the length of the main craze where, in the low velocity experiments, it stops; the local stress drops below the craze propagation stress. As it propagates forwards from its point of origin the craze also extends sideways at a similar speed because a high transverse stress parallel to the direction of crack propagation persists until the main crack tip has passed by. This process can be seen in Fig. 15 which shows the propagating craze wedge viewed in monochromatic light (λ = 546 nm) from a direction normal to the plane of the craze. The black triangle is the branching craze, O is its origin and the lines, L, are the loci of its intersection with the main craze layer.

In the HMW–PMMA used in one series of experiments advance fractures occurred only very occasionally in the slow velocity region of crack propagation (velocity c about 0.1 m sec^{-1}) where they probably initiate at "strong" heterogeneities such as dirt particles. In the higher velocity region (c about 100 m sec^{-1}) the advance fractures cover the surface of fracture with a frequency which increases the crack velocity [13]. The nucleation sites in this case are evidently from a different source as they are more numerous and more uniformly distributed. The increasing density of advance fractures could arise from the increase in the near crack tip stress with rising crack velocity. Alternatively, it could be a geometric effect arising from an increase in the craze thickness with velocity; more nucleation sites are "captured" by a thicker craze. In the fast fracture region the interference colours arising from the craze layer on the fracture surface increase from second order to fourth order colours in the region C–D (Fig. 8).

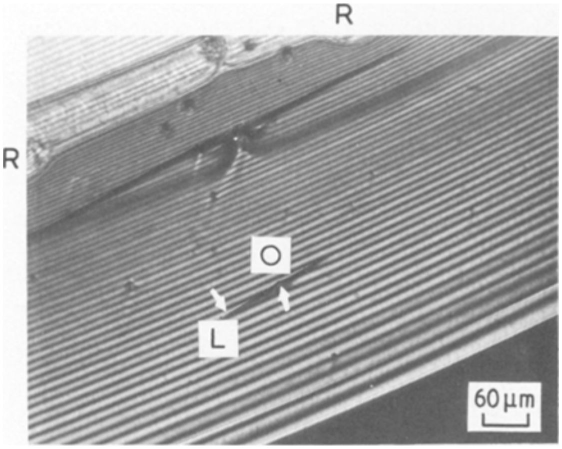

Figure 15 Craze wedge preceding the crack tip, R, of a cleavage crack in polystyrene; viewed in reflected light (λ = 546 nm) from a direction normal to the plane of the craze-crack. The forward edge of the craze is at L, a branch craze has initiated at O.

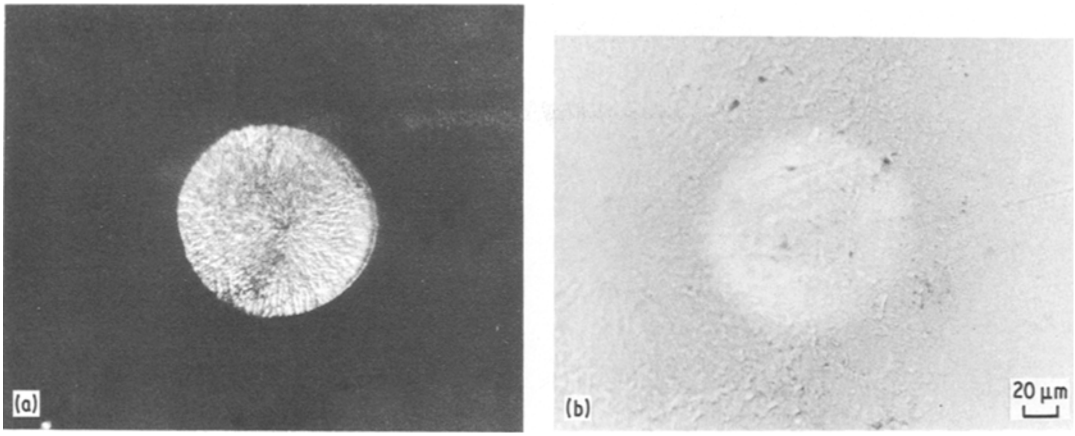

Figure 16 A subsurface fracture nucleated in a branching craze found in the region C (see Figs. 7 and 8); (a) oblique illumination and plane of focus on the subsurface crack, (b) normally incident reflected light and focus on the main fracture surface.

The branching crazes shown in Figs. 9 and 10 begin at a position 5.5 cm from the tip of the saw cut where, from Fig. 8, the crack speed is about 420 m sec^{-1}. This must correspond to some critical value of the craze stress. The branches occur roughly periodically in the zones B_1, B_2, B_3, and B_4 and their spacing, about 3×10^{-3} m, is the same as that of the rough-smooth bands which immediately follow on the surface of fracture at E. Each zone, which spans the specimen thickness (the width of the crack front), consists of several branching crazes each nucleated at a point from which they spread out sideways a distance of about 5×10^{-4} m and frequently overlapping. The branching crazes in the high velocity region propagate at an angle of about 20 degrees to the main surface of fracture. This results from the very high forward velocity of the stress field around the moving crack-craze tip region relative to the nucleated branching craze.

As the speed of the main crack increases further, secondary cracks are nucleated in the branching crazes, usually near their forward tips at their furthest depth below the main surface of fracture but not connected with it (Fig. 16). At slightly higher crack speeds the microcracks propagated backwards to join up with the main fracture surface. At this stage the bands become clearly visible on the fracture surface. The plug of bulk polymer between the branch craze and the main craze may be pulled out leaving a protrusion on one surface of fracture and a steep sided hole on the other. In one of the earliest published descriptions of the bands in HMW–PMMA, Zandman [14] aptly described this feature as "fractures by pulling out" (translated). The branching crazes are initiated independently at several points along the crack front from which they spread out laterally even though the craze layer in which they nucleate is itself a continuous layer. For this reason the boundary line between the rough band and the smooth region preceding it is not a sharp continuous line as is usually seen with polystyrene bands (Fig. 1). Another reason is that the large number of advance fractures nucleated in the propagating craze wedge must produce an irregularity in the line of the crack front [15] and possibly in the craze tip also so that the line of the maximum stress would not be straight across the crack front. This type of fracture can also result from the intersection of one branching craze with another which is initiated just ahead of it, as shown in the micrograph shown in Fig. 17. The sudden onset of craze branching and the consequent increase in crack propagation energy is consistent with the observation of a "break point" in the variation of the dynamic stress intensity factor with crack speed at 396 m sec^{-1} [16].

In an interesting observation, Cotterell [13] has found that while the distance between the foci and the tips of the fracture parabolae is less than 25 μm at crack velocities below 100 m sec^{-1} or so, in almost half the advance fractures this distance is between 50 and 100 μm at speeds above 400 m sec^{-1}. At very low crack velocities and at room temperature, the craze layer on the fracture surface shows third order interference colours and it is known that the propagating craze layer is, for

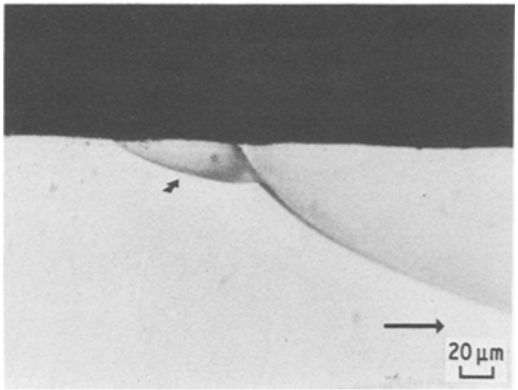

Figure 17 Intersection of successive branching crazes forming block of polymer (arrow) which may be pulled from the fracture surface. Reflected light was used.

these conditions, about 25 μm in length [17]. In the high velocity region the fracture surface interference colours increase with velocity from second to fourth order, so that it is unlikely that the length of the craze wedge under these conditions could be more than 50 μm. If this is the case, it follows that many of the advance fractures in the high velocity region are indeed nucleated in the bulk polymer ahead of the main craze wedge [5]. But this effect first occurs beyond where the craze branching begins which would seem to preclude independent nucleation of secondary crazes and microcracks ahead of the main craze-crack as the cause of the bands when they first appear.

The origin of the rough bands in HMW–PMMA is the nucleation of branching crazes at some critical crack speed which, at higher stress levels, are the sites for secondary crack nucleation above and below the level of the main fracture surface. The band of multiple cracking and crazing causes a deceleration of the fracture and a reversion to the simpler mode of fracture followed by acceleration of the crack and the nucleation of the next band.

4. Discussion
4.1. Wallner lines

It has been suggested [16, 18] that the bands are a Wallner line phenomenon arising from the interaction of the propagating crack with shear waves generated at the side surface of the specimen. Suppose that a straight crack front propagates through the specimen and at some position R_1 (Fig. 18) a shear wave generated at the surface discontinuity, D, spreads out at the wave velocity (about 1000 m sec^{-1} in PMMA [16]); if the crack velocity remains constant, the locus of the perturbation where the shear wave intersects the moving crack front is an oblique line D–W. At the crack tip the superposition of the shear from the stress wave produces an increase in the maximum tensile stress ahead of the crack tip and a rotation of the principal stress direction. In the simplest cases the crack deviates from its preceding direction, returning to its original plane after the stress wave has passed, leaving a slight ripple on the surface of fracture. It has been proposed [16] that this effect, the transient stress increase, is the cause of the bands in PMMA by the nucleation of crazes, at several different levels above and below the plane of the craze-crack, ahead of the main crack-craze tip [18]. It is evident that in any mechanism of fracture which occurs at some critical stress, the "pattern" of crack nucleation can be controlled by propagating stress waves. However, the characteristic cross hatch pattern of the bands observed on the HMW–PMMA fracture surfaces also occurs on the high velocity surfaces of fracture in polystyrene (Fig. 19) (Type II bands described above) where it is known that the stress waves are not an essential factor in their mechanism of formation. In poly-

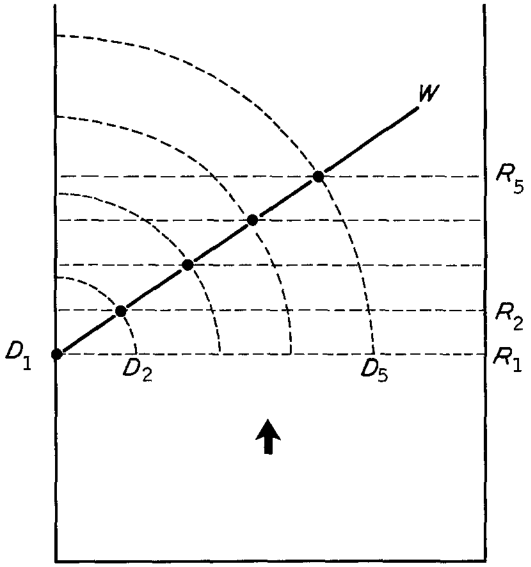

Figure 18 Schematic diagram of the formation of fracture bands oriented obliquely to the direction of crack propagation by the interaction of a stress wave, originating from point D_1, with the crack front at R_1. D–W marks the locus of the point of interaction of the crack and the stress wave as they continue to propagate.

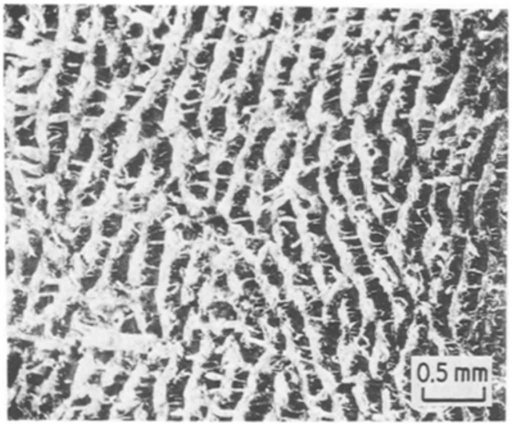

Figure 19 Cross hatch pattern of fracture bands on the fracture surface of polystyrene using dark field illumination.

styrene the bands can be formed under quasistatic conditions where stress waves are not likely to be important. For this reason the appearance of the cross hatch pattern is not in itself a compelling reason to assume that stress waves are a necessary component for the formation of the bands in HMW–PMMA. The latter would still appear even in the absence of stress waves, if the specimen boundaries were far enough away from the crack front for example, and provided the crack tip stress was high enough to initiate the craze branching mechanism described above. In the region of the bands the crack velocity oscillates, and in the smooth region where it is increasing in speed it is just below the critical state where craze branching and shattering will occur. The relatively small stress amplitude of the shear wave is sufficient to control when and where the critical stress is reached irrespective of what the actual mechanism is. Here we propose that it is based on crack branching rather than nucleation of crazes ahead of the main propagating craze-crack front, although this latter effect may become operative at still higher speeds as an adjunct to craze branching.

4.2. The Yoffe effect

Kusy *et al.* describe a "stick-slip" mechanism [19, 20]: "sticking occurs after the sudden release of strain energy near the crack tip following the formation of a shower of microcracks . . . slipping occurs as the crack tip subsequently follows a craze and then accelerates". The cause of the shower of microcracks they seem to associate with the Yoffe effect [21], the change in stress distribution around the crack as the velocity increases. This and other analyses [22] assume a steady state propagation of the crack at constant velocity and length. The maximum in the polar stress, $\sigma_{\theta\theta}$, which is directly in front of the crack tip at zero velocity, moves out to about 60° on either side of the plane of the crack velocity of about 0.6 C_2. However, as was pointed out by Williams [23] and others [24], the radial stress, σ_{rr}, is larger than the polar stress at all velocities. Williams [23] also showed that even in the case of a static crack the maximum principal tensile stress at a given radial distance from the crack tip does not occur directly ahead of, but at an angle of 60° to the plane of the crack at which position the maximum principal stress direction is perpendicular to the plane of the crack [23] (and parallel to the applied stress). So that even in the case of an ideal elastic crack it would be difficult to draw conclusions concerning the nucleation sites of secondary microcracks. For a glassy polymer where the crack is preceded by the craze layer the distribution of stress will be different, but it seems reasonable to assume that the dynamic stress field will be distorted in a way qualitatively similar to that predicted by Yoffe. There is an additional complication arising from the heat dissipated at the crack tip. In the case of PMMA, increases in crack tip temperature have been variously measured and estimated to be several hundred degrees Kelvin [25, 26] which would generate a compressive thermal stress contribution around the crack tip [27].

While the above mentioned dynamic stress analysis may explain the curvature of the paths of cracks running at high speed, they do not contain any explicit mechanism for the initiation of branching or nucleation of secondary microcracks (crazes).

4.3. Energy criteria for crack branching

In a plate containing an edge crack and loaded in tension until the crack just begins to propagate, and thereafter, assuming a fixed grips condition, then, as the crack length increased the energy available beyond that of the surface fracture work of the crack and the dissipative processes associated with its formation accelerates the crack. But the potential energy is only available from within a circular region of radius C_1t, where C_1 is the dilatational wave velocity and t is the time for which the crack has been in motion. Assuming that the fracture surface work is independent of the crack velocity, the crack speed should reach a maximum at the Rayleigh (surface) wave velocity, C_r, which is about 0.93 C_2 for a material with a Poisson's ratio of 1/3 and where C_2, the shear wave velocity, is about 1000 m sec^{-1} for HMW–PMMA [18].

The strain energy release rate for the moving crack is of the form [28, 29]

$$G = g(v)\, G^* \qquad (2)$$

where g(v) is a velocity factor which decreases approximately linearly from unity at zero (Fig. 20) velocity to zero at the velocity of the Rayleigh wave and G^* is a "static" strain energy release rate which depends on the current crack length, the applied load and the history of crack extension but not on the current crack velocity. In general, G^* would be less than the true static strain energy release rate, G_s, for the same load and crack length. An equivalent relationship for the stress intensity factor which describes the amplitude of the stress field around the crack tip is

$$K = k(v)\, K^* \qquad (3)$$

where the terms are defined analagously to those in Equation 3 above. The equation contains no explicit dependence of G on the crack acceleration so that the propagating crack behaves as an inertialess disturbance; the crack velocity changes instantaneously in response to a change in the driving force [28].

From this it follows that for a simple bifurcation both cracks cannot propagate unless the velocities of both branches decrease instantaneously to the extent that g(v), the velocity factor, doubles. This first becomes possible when the crack velocity at bifurcation is above about 0.5 C_r. For HMW–PMMA, however, the fracture surface work increases with velocity so that at branching, if the new velocity is reduced the fracture surface work is also, and bifurcation would be possible at lower crack speeds. Thus an essentially brittle elastic fracture under a fixed grips condition can only bifurcate above some critical velocity which is likely to be a significant fraction of the theoretical terminal velocity. In HMW–PMMA macroscopic branching occurs at about 700 m sec^{-1} and the surfaces of fracture of these branches are at first banded in the manner characteristic of a crack speed of about 400 m sec^{-1}.

An energetically based criterion for crack branching has been proposed as an explanation for the formation of bands in polystyrene as well as in PMMA [30] (although the molecular weight of the PMMA was not specified, see above). But the specific fracture surface energy must, on the scale of the microstructure of the bands at least, vary by perhaps an order of magnitude as the mechanism of craze rupture changes with velocity. Under these conditions a detailed and rigorous analysis of the crack propagation dynamics would be quite complex. In any event the simple criterion for crack bifurcation proposed by Friedrich [30] cannot explain the essence of the banding phenomenon in polystyrene as it occurs in the simplest case shown in Fig. 1; the Type I bands do not involve any form of branching. The crack propagation is a process of discontinuous self replication of the craze-crack tip by the successive formation and rupture of craze wedges. And furthermore, a criterion for initiation of shatter bands based on the energetics of crack branching cannot explain why the Type II bands start immediately under quasi-static conditions with multiple crazing and cracking and not simple bifurcation.

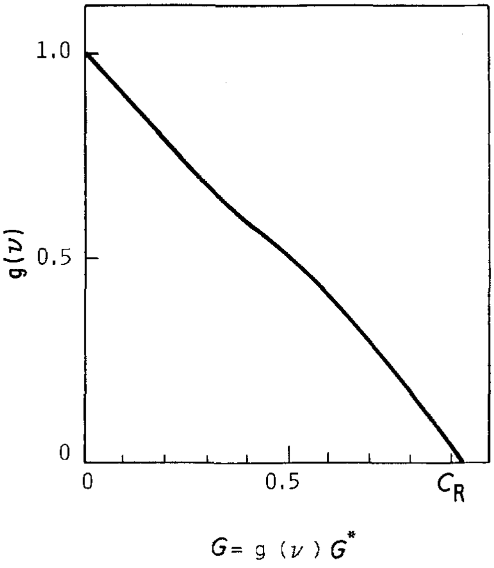

Figure 20 Variation of the velocity factor, g(ν), of the strain energy release rate with crack velocity.

4.4. Molecular weight and band spacing

Since the initiation of the bands is a velocity effect, the rib or band spacing observed on any specimen will depend on the history of the acceleration of the crack (with respect to the crack length) along the surface of fracture. In the specimens discussed above which contained a 0.6 cm "pre-crack", the bands have a spacing at their beginning ten times larger than that of the beginning bands in specimens of Kusy *et al.* [19] (2.5 cm wide strips, 0.25 cm saw cut). As the crack accelerates across the specimen, the stress at its tip increases continuously (ignoring the momentary accelerations and decelerations associated with the bands) so that the bands tend to include a shatter zone of increasing depth and length and this, as pointed out by Kusy *et al.* [19], increases the rib spacing. In our specimens the very rough bands, immediately preceding macroscopic branching, are several times larger than the widest bands observed by Kusy [19]. The variations of rib spacing with molecular weight and temperature [31, 32] would reflect the dependences of the craze branching stress, the craze and crack nucleation stresses and the craze fracture stress on these variables all at high strain rates. In general, if the applied stress on the specimen at fracture decreases so that the stored elastic energy is lower, the bands should tend to be less coarse and closer together.

4.5. Crack branching in nonpolymeric glasses

The actual mechanism by which cracks bifurcate has been most extensively investigated in the case of glass. Two possibilities have been described. In the first [33, 34] the propagating crack front is deflected from its plane at some point along its length by a microstructural defect. This tendency will be greatest at high velocities where the polar stress becomes nearly constant over a wide arc ahead of the crack tip (the Yoffe effect). In order to form a macroscopic branch, the deviated sector of the crack front must separate and spread sideways as well as forwards from its point of origination, linking up perhaps with other similar branches nucleated at other points along the front. It is also necessary that the branch crack grows away from the main crack (or the other fork) and not back towards it, as would be more likely at lower crack velocities. In building up to the condition for the macroscopic bifurcation, the embryonic branches at the crack front can be nucleated at numerous points and then run back into the main fracture, but they cannot develop into proper branches for either energetic or stress reasons. The diversion of energy into these "subcracks" is small and has a barely detectable effect on the fracture velocity [35] which continues to increase gradually just as the surface of fracture continuously increases in roughness until a macroscopic bifurcation becomes possible.

In the other mechanism, independent, secondary microcracks are nucleated ahead of the main crack front at structural heterogeneities. For the purposes of the present discussion, the distinction between these two possibilities is not imporant. In either case, the mechanism for initiation of branch cracks is dependent on dynamic effects (Yoffe) to overcome the tendency for secondary cracks to merge back with the main crack front. This is only significant at $\dot{c} > 0.5\ C_2$ at which speed the energetics are probably already favourable to branching.

In glass each branch starts with a smooth mirror region, accelerates to "mist" then "hackle" [35] and branches again. Sometimes this sequence of macroscopic branching in glass gives a semblance to the "micro" banding in glassy polymers [36]. In the former case, however, each "band" corresponds to a macroscopic crack branching so that the fracture surfaces are not coplanar as is the case for the bands on the polymer fracture surfaces.

5. Conclusion

The mechanism of craze branching in the polymers depends on two factors: the formation of a defect in the craze, an "advance fracture", and a critical stress in the direction parallel to the plane of the craze. Because of the plasticity of the craze matter and the sharpness of the craze-bulk boundary (thickness 5 nm or less) the craze defects are probably uniform in their mechanical stress concentration effect on the adjacent bulk so that under conditions where craze defects are plentiful the branch craze nucleation depends primarily on the attainment of a critical stress at the crack tip and thus a critical crack velocity. This mechanism does not depend explicitly on the Yoffe effect. The stress in and around the craze may increase simply as a result of the increasing strain rate at the crack tip and could, in principle, reach the critical level at crack speeds where the Yoffe effect is negligible. There is, in addition, the further complication of the nucleation of a crack in the branch craze before a branching crack is produced. At first the branch is a propagating craze for which the work of extension would increase as its length. Because of the symmetry of the craze wedge the branching crazes are equally likely to be nucleated on both craze–bulk boundaries. Each branch craze spreads sideways and forwards from its point of origination with a velocity comparable to that of the main craze, linking up or overlapping with adjacent branching crazes. In effect the craze trifurcates. The separate craze branches are initiated by a critical stress condition, but they may not propagate as independent crack branches for two reasons. Firstly, the local stress level must be further increased until cracks are nucleated in the branches and secondly, the energy condition must be satisfied. Prior to this last condition, the initiation of the branch crazes inevitably decelerates the main crack so that the velocity is then below the critical level and branching crazes cannot be initiated. The fracture then propagates as a single craze-crack until the velocity has increased to its critical level once again. The oscillation in fracture mechanism gives rise to the rough-smooth bands which continue until the dynamic energy condition is satisfied and macroscopic branches can propagate.

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